WOPA 2016 - Workshop on Orthogonal Polynomials and Applications

Analysis Area - Center of Mathematics of University of Porto (CMUP)

Mathematics Department of Faculty of Sciences of University of Porto

Porto, February 17th, 2016

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- Mathematics Department of Faculty of Sciences of University of Porto

The **WOPA 2016 - Workshop on Orthogonal Polynomials and Applications** is a one day meeting organized by Analysis Area of Center of Mathematics of University of Porto (CMUP) and will be held at Mathematics Department of Faculty of Sciences of University of Porto.

This event takes place following the past three meetings held in Porto in 2003, 2006 and 2007, that gathered PhD students and supervisors members of CMUP working in the field of orthogonal polynomials, Padé approximation, linear systems, special functions and integral transforms.

After an interruption of several years, we had the idea to resume this meeting back to gather the members of the Analysis Area of CMUP and other researchers working in the above mentioned topics.

This workshop is constituted by eight lectures of thirty minutes each and is divided into Part I and Part II. The first part is devoted to some Theoretical Aspects of Orthogonal Polynomials. Part II deals mostly with Integral Transforms and Special Functions, and also with Padé approximation, that is an important Application of Orthogonal Polynomials. In each part, we will have four presentations.

The organizer,

Zélia da Rocha

Location: Room FC1.007, Maths Building

Shedule: Wednesday, February 17th

14h	**Welcome**	
14h 20m	**Opening**	
	Part I	
14h 30m	Zélia *da Rocha*	DM-FCUP
15h	Ana Filipa *Loureiro*	Univ. Kent
15h 30m	*Ângela Macedo*	DM-UTAD
16h	Teresa *Mesquita*	IPVC
16h 30m	**Coffee break**	
	Part II	
17h	José Luís *Cardoso*	DM-UTAD
17h 30m	Semyon *Yakubovich*	DM-FCUP
18h	Sílvio *Gama*	DM-FCUP
18h 30m	João Emílio *Matos*	DM-ISEP
19h	**Dinner** at Madureira's	

Programme

- **14h Welcome** • **14h 20m Opening Session**
- **14h30m - 15h**
  Zélia da Rocha, University of Porto
  Perturbed Chebyshev polynomials
- **15h - 15h30m**
  Ana Filipa Loureiro, University of Kent, U.K.
  On d-orthogonal polynomials and an alternative discrete Painlevé I equation
- **15h30m - 16h**
  Ângela Macedo, University of Trás-os-Montes e Alto Douro
  Chebyshev Polynomials via quadratic decomposition of the canonical sequence
- **16h - 16h30m**
  Teresa Mesquita, Instituto Politécnico de Viana de Castelo
  Applying general cubic decompositions to 2-orthogonal polynomial sequences and to the canonical sequence
- **16h30m - 17h Coffee Break**
- **17h - 17h30m**
  José Luís Cardoso, University of Trás-os-Montes e Alto Douro
  Hölder and Minkowski's inequalities for the q and the (q, ω) integrals. q and (q, ω) analogues of the Lebesgue spaces
- **17h30m - 18h**
  Semyon Yakubovich, University of Porto
  The Kontorovich-Lebedev transform and its application to the Bernoulli, Euler numbers and Riemann zeta-values
- **18h - 18h30m**
  Sílvio Gama, University of Porto
  Around eddy viscosities and Padé approximants
- **18h30m - 19h**
  João Emílio Matos, Instituto Superior de Engenharia do Porto
  On location of Froissart doublets of Padé approximants from orthogonal expansions
- **19h Dinner at Madureira's**

Abstracts - Part I

- *Perturbed Chebyshev polynomials*

Zélia da Rocha

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Abstract:

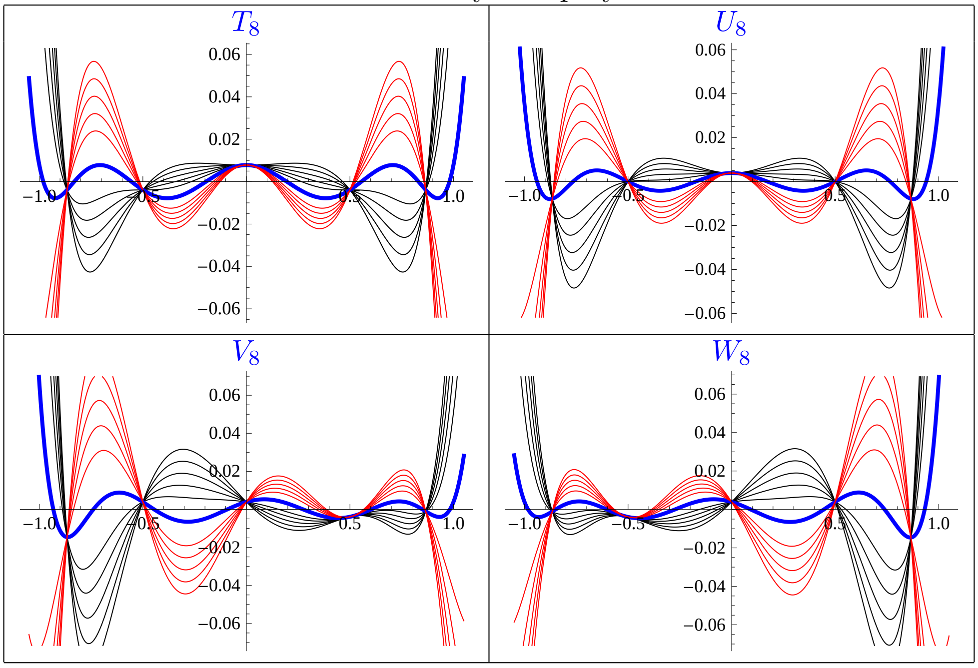
In some applications one is led to consider perturbations of orthogonal polynomials translated by a modification on the first coefficients of the second order recurrence relation satisfied by these polynomials. This transformation can promote a deep change of properties; nevertheless there is a large set of forms that are preserved by perturbation: the second degree forms [7,8]. Moreover, second degree forms are also semi–classical. Among the classical forms (Hermite, Laguerre, Bessel and Jacobi) only certain Jacobi forms are of second degree [5], including the four Chebyshev forms [4]. Thus, it is important to clarify and explicit the properties of perturbed Chebyshev polynomials. From the point of view of perturbation, the Chebyshev form of second kind is the most simple and the other three forms of Chebyshev can be considered as perturbed of it [1,2].

By means of a new general method and the corresponding symbolic algorithm *PSDF* [1,2] based on Stieltjes equations [7,8], we are able to explicit several semi–classical properties of perturbed second degree forms, namely: the Stieltjes function, the Stieltjes equation, the functional equation, the class, a structure relation and the second order linear differential equation as well as the first moments and the generating function of perturbed forms. Applying the algorithm *PSDF* to the Chebyshev form of second kind, we achieve to explicit the above mentioned properties for perturbations of several orders [1,2] generalizing existent results in literature [3,6]. From these properties, we can easily derive similar ones for the other three forms of Chebyshev.

Key words: Chebyshev polynomials; second-degree forms; differential equations; symbolic computations.

2010 Mathematics Subject Classification: 34, 33C45, 33D45, 42C05, 33F10, 68W30, 62-09, 33F05, 65D20, 68-04.

Perturbed Chebyshev polynomials



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- *On d-orthogonal polynomials and an alternative discrete Painlevé I equation*

Ana Filipa Loureiro

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University of Kent

Abstract:

I will talk about certain d-orthogonal polynomials whose recurrence coefficients can be described via an alternative discrete Painlevé I equation (dP-I). Furthermore, I will discuss some thrilling properties of special solutions of dP-I.

- *Chebyshev Polynomials via quadratic decomposition of the canonical sequence*

Ângela Macedo

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Abstract:

We give the general quadratic decomposition of the sequence of monomials $\{x^n\}_{n\geq 0}$ and prove how the correspondent component sequences are connected with Chebyshev polynomial sequences. The two-way street that joins together the polynomial sequences brought by quadratic decomposition and those famous polynomial sequences is established by the reversed polynomials and initial perturbations.

References and Literature for Further Reading

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- *Applying general cubic decompositions to 2-orthogonal polynomial sequences and to the canonical sequence*

Teresa Mesquita

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Abstract:

The single definition of a 2-symmetric sequence raises a special case of a full polynomial cubic decomposition, firstly introduced for orthogonal sequences in 1989 by Rodriguez and Tasis, and more recently, in 2011, completely generalized by Maroni, Mesquita and da Rocha.

The application of a general cubic decomposition to a polynomial sequence yields up to nine further polynomial sequences which have been already computed and carefully studied for some given families of polynomial sets. After a short overview of the subject, we will present a few of the known characteristics of the components obtained when we decompose (cubically) 2-orthogonal polynomial sequences. A particular attention will be given to the interesting output of the cubic decomposition of the canonical sequence $\{x^n\}_{n=0}^{\infty}$.

Abstracts - Part II

- *Hölder and Minkowski's inequalities for the q and the (q,ω) integrals. q and (q,ω) analogues of the Lebesgue spaces*

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Abstract:

For $0 < q < 1$, $\omega \geq 0$, $\omega_0 := \omega/(1-q)$, and I a set of real numbers, the Hahn operator acting on a function $f : I \to \mathbb{R}(\mathbb{C})$ is defined by

$$D_{q,\omega}[f](x) := \frac{f\big(qx+\omega\big) - f(x)}{(q-1)x+\omega}\ , \quad x \in I \setminus \{\omega_0\}\ .$$

Its inverse operator is given in terms of the so-called Jackson-Thomae $(q,\omega)-$integral, also called Jackson-Nörlund $(q,\omega)-$integral. For $\omega = 0$ one obtains the Jackson's $q-$operator, D_q, whose inverse operator is given in terms of the so-called Jackson $q-$integral. We settle conditions that guarantee the Hölder and the Minkowski's inequalities for the $(q,\omega)-$integral. By establishing links between $D_{q,\omega}$ and D_q, as well as between the q and the (q,ω) integrals, we show how to obtain the properties of $D_{q,\omega}$ and the $(q,\omega)-$integral from the corresponding ones fulfilled by D_q and the $q-$integral.

We also consider $(q,\omega)-$analogues of the Lebesgue spaces.

These results were motivated by our previous research works on basic Fourier series.

Keywords: Jackson $q-$integral, $q-$analogues, Jackson-Nörlund $(q,\omega)-$integral, $(q,\omega)-$Lebesgue spaces.

AMS Classification: 33E20, 33E30, 40A05, 40A10.

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- *The Kontorovich-Lebedev transform and its application to the Bernoulli, Euler numbers and Riemann zeta -values*

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Abstract:

Various new identities, recurrence relations, integral representations, connection and explicit formulas are established for the Bernoulli, Euler numbers and the values of Riemann's zeta function $\zeta(s)$. To do this, we explore properties of some Sheffer's sequences of polynomials related to the Kontorovich-Lebedev transform.

- *Around eddy viscosities and Padé approximants*

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Abstract:

In this presentation we show the efficiency of Padé approximants combined with Mathematica to resume expansion of the eddy viscosity expressed in inverse powers of molecular viscosity for two-dimensional, incompressible, parity-invariant and six-fold-rotation-symmetric flow.

We also present an algorithm which allows us to compute the inverse of the two-dimensional Laplacian operator restricted to periodic functions.

- *On location of Froissart doublets of Padé approximants from orthogonal expansions*

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Abstract:

We present some numerical results about the location of Froissart doublets of Padé approximants from perturbed orthogonal expansions. These results are a generalization of the Froissart's numerical experiments with power series. Our results suggest that the Froissart doublets of Padé approximants are located, with probability one, on the Joukowski transform of the natural boundary of the random power series.