On a heat kernel for the index Whittaker transform

M.M. Rodrigues[‡] and S. Yakubovich[†]

[‡]Department of Mathematics, University of Aveiro
Campus Universitário de Santiago, 3810-193 Aveiro, Portugal
E-mail: mrodrigues@ua.pt

[†]Department of Mathematics, Faculty of Science, University of Porto
Rua do Campo Alegre 687, 4169-007 Porto, Portugal
E-mail: syakubov@fc.up.pt

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Abstract

We introduce a notion of the heat kernel associated with the index Whittaker transform. We study its differential and mapping properties and a relationship with a family of the corresponding Weierstrass's type transforms.

Keywords: Index Whittaker transform, Whittaker function, modified Bessel function, heat kernel

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1 Introduction

In this paper, we deal with the index Whittaker transform

$$\mathcal{W}_\mu[f](\tau) = \lim_{N\to+\infty} \int_{1/N}^{N} x^{-2} W_{\mu,i\tau}(x) f(x)\, dx, \tag{1}$$

where $W_{\mu,i\tau}(x)$ is the Whittaker function, with $x,\tau \in \mathbb{R}_+$, $\mu < \frac{1}{2}$. The convergence of (1) is guaranteed in the sense of the norm in $L^2\left(\mathbb{R}_+, \frac{1}{\pi^2}\left|\Gamma\left(\frac{1}{2}-\mu+i\tau\right)\right|^2 \tau\sinh(2\pi\tau)d\tau\right)$ (see [8]). Precisely, the operator

$$\mathcal{W}_\mu[f] : L^2\left(\mathbb{R}_+, x^{-2}dx\right) \to L^2\left(\mathbb{R}_+, \frac{1}{\pi^2}\left|\Gamma\left(\frac{1}{2}-\mu+i\tau\right)\right|^2 \tau\sinh(2\pi\tau)d\tau\right)$$