Fractional two-parameter Schrödinger equation

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Abstract

This work is intended to investigate the multi-dimensional space-time fractional Schrödinger equation of the form

$$\left({}^C D^\alpha_{t_0^+} u\right)(t,x) = \frac{i\hbar}{2m}\left({}^C\nabla^\beta u\right)(t,x)$$

with $\hbar$ the Planck's constant divided by 2π, m is the mass and $u(t,x)$ is a wave function of the particle. Here ${}^C D^\alpha_{t_0^+}, {}^C\nabla^\beta$ are operators of the Caputo fractional derivatives, where $0 < \alpha \leq 1$ and $1 < \beta \leq 2$. The wave function is obtained using Laplace and Fourier transforms methods and a symbolic operational form of solutions in terms of the Mittag-Leffler functions is exhibited. It is presented an expression for the wave function and for the quantum mechanical probability density. Using Banach fixed point theorem, the existence and uniqueness of solutions is studied for this kind of fractional differential equations.

Keywords: Fractional partial differential equations, Radial Fourier transform, Mittag-Leffler function, Time-dependent Schrödinger equation, Fixed-point theorem.

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