On the optimal control of a cascade of hydro-electric power stations

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Abstract

Water is becoming a scarce resource and its use has attained, in more advanced countries, a certain degree of sophistication. This has had impact also in the way water is used to produce electric energy specially if there is a possibility of reusing the downstream water and there is a situation of drought even if mild. This may be implemented in modern reversible hydroelectric power stations, associated with reservoirs along a river basin with a cascade structure, where it is possible both to turbine water from upstream to produce electric power and to pump from downstream to help to refill an upstream reservoir. Here we present a possible model for a cascade of four hydro-electric power stations where two of the stations have reversible turbines. There are constant restrictions on the water level and water volumes of the reservoirs, and on specified river inflows that are time functions; market prices of the produced electric energy and flow rates which are also time functions. The objective is to optimize the profit of producing power providing at the same time some guidelines on when, how much and in what direction to allow the water flow, whenever possible. The situation is modeled as an optimal control problem, solved iteratively using derivative free numerical methods. Even with software that was not written specifically for the situation, the problem was solved with realistic data. The results are promising enough to encourage a more ambitious process of finding better software to solve the much bigger real problem faced by an energy producing enterprise (REN).

Keywords: Capacity Planning, Energy Policy and Planning, Enterprise Resource Planning Systems, Environmental Management Facilities Planning and Design, Natural Resources, Production, Variational Problems.

1 Introduction

Water is becoming a scarce resource and its use has attained, in more advanced countries, a certain degree of sophistication. This has had impact also in the way water is used to produce electric energy and for some time the operation of multireservoir systems has attracted the attention of those responsible for its rational management and operational decisions (Labadie, 2004; Ladurantaye et al, 2009). This is especially important if there is also a possibility of reusing the downstream water and there is a situation of drought even if mild. This may be implemented in modern reversible hydroelectric power stations, associated with reservoirs along a river basin with a cascade structure, where it is possible both to turbine water from upstream to produce electric power and to pump from downstream to help to refill an upstream reservoir. Here we present a model for a cascade of hydro-electric power stations where some of the stations have reversible turbines. There are constant restrictions on the water level and water volumes of the reservoirs, and on specified river inflows that are functions of time; market prices of the produced electric energy and flows which are also functions of time. The objective is to optimize the profit of producing power providing at the same time some guidelines on when, how much, and in what direction to allow the water flow, whenever possible. The problem is considered in the framework of a discrete-time optimal control problem and is solved using numerical methods. The simulation uses real data from Rede Eléctrica Nacional (REN) and is a development of a basic model already studied at REN.

The paper is organized as follows: in section 2, the problem is stated and the model is presented; in section 3 the computational experience is described and presented the results obtained; section 4 contains some conclusions.

2 Problem Statement

Consider the following discrete-time optimal control problem with mixed constraints. The functional has the form

$$P(q,s,V^{in}) = \int_0^T price(t)\left(\sum_{i=1}^{I} r_i(t)\right) dt$$

where $V^{in} = (V_1^{in}, \ldots, V_I^{in})$ is the vector of initial stored water volumes in the reservoirs $i = 1, \ldots, I$; $q(t) = (q_1(t), \ldots, q_I(t))$ and $s(t) = (s_1(t), \ldots, s_I(t))$, $t = 1, 2, \ldots, T$, are the controls, representing the turbined/pumped volumes of water and spillways for each reservoir at time t. The functions $r_i(t), i = 1, 2, \ldots, I$, are given by

$$r_i(t) = \begin{cases} 9.8 * q_i(t) * \left(h_i(t) - \Delta h_i^T(t)\right) * \mu_i^T * (1-\phi_i) \;\; if\; q_i(t) \geq 0 \\ 9.8 * q_i(t) * \left(h_i(t) - \Delta h_i^P(t)\right) * 1/\mu_i^P * (1-\phi_i) \;\; if\; q_i(t) < 0 \end{cases}$$

where the $h_i(t)$ are the differences in water levels (see Figure 1) and $\Delta h_i(t)$ are head losses. The functions $r_i(t)$ connect the amounts of turbined water and the values of the gross head. The dynamics

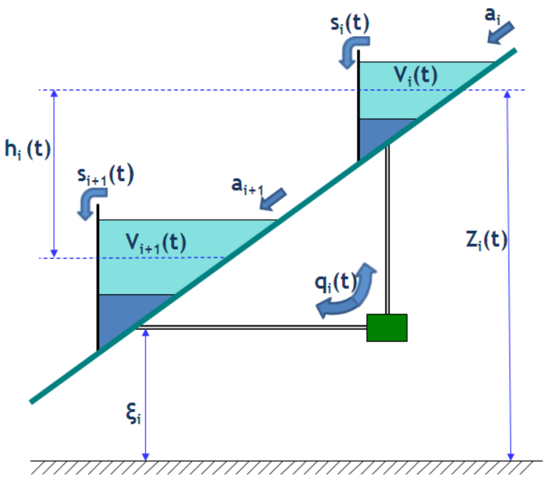

Figure 1: Two cascade reservoirs.

of water volumes in the reservoirs, $V_i(t)$, $i = 1, 2, \ldots, I$, is described by the following discrete-time control system

$$V_i(t) = V_i(t-1) + a_i - q_i(t) - s_i(t) + \sum_{m \in M_i} q_m(t) + \sum_{n \in N_i} s_n(t), \;\; t = 1, 2, \ldots, T, \;\; i = 1, 2, \ldots, I,$$

$$V_i(0) = V_i^{in} \;\; i = 1, 2, \ldots, I.$$

where M_i represents the set of reservoir indices from which comes the water flow to reservoir i , from pumping or turbining and N_i is the set of reservoir indices contributing to the spillway from reservoir i. Moreover, the controls and the water volumes satisfy the following constraints

$$h_i(t) = Z_i^0 + \alpha_i \left(\frac{V_i(t)}{V_i^0} - 1\right)^{\beta_i} - \max\left\{ Z_{i+1}^0 + \alpha_{i+1}\left(\frac{V_{i+1}(t)}{V_{i+1}^0} - 1\right)^{\beta_{i+1}}, \xi_i \right\},$$

$$Z_i(t) = Z_i^0 + \alpha_i \left(\frac{V_i(t)}{V_i^0} - 1\right)^{\beta_i},$$

$$
\begin{aligned}
&\zeta_i\left(h_i(t)-h_i^0\right)-q_i^{0P}\le q_i(t)\le q_i^{0T}\left(h_i(t)/h_i^0\right)^{\frac{1}{2}}\ ,\\
&Z_i^{min}\le Z_i(t)\le Z_i^{max}\ ,\\
&V_i^{in}-a_i\le V_i(T).
\end{aligned}
$$

Here $V_i^0,\ i=1,2,\dots,I$ are the minimal water volumes; $Z_i(t),\ i=1,2,\dots,I$ are the water levels in the reservoirs; Z_i^0, Z_i^{min}, and Z_i^{max} stand for nominal, minimal and maximal water levels (meters above sea level) respectively; $h_i^0,\ i=1,2,\dots,I$ are nominal heads, and $\xi_i,\ i=1,2,\dots,I$ are tailwater levels; $q_i^{0T},\ i=1,2,\dots,I$ and $q_i^{0P},\ i=1,2,\dots,I$ are the nominal turbined and pumped water volumes; $a_i,\ i=1,2,\dots,I$ are the incomming flows; finally $\alpha_i,\beta_i,\zeta_i,\ i=1,2,\dots,I$ are positive constants. The optimal values $V_i^{in},\ i=1,2\dots,I$, give the mean volumes of water that are necessary to keep in the reservoirs when the incomming flows are $a_i,\ i=1,2,\dots,I$.

A typical one day price function, $price(t)$, is shown in Figure 2. It should be noted the high variability

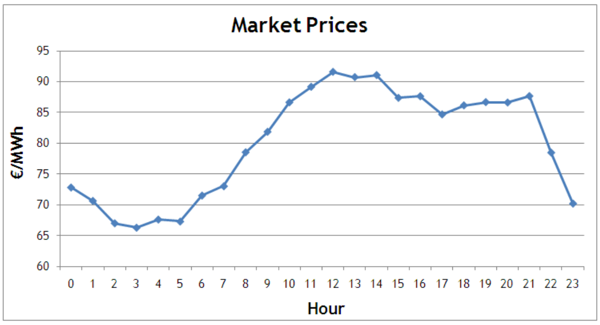

Figure 2: One day real market prices of electricity.

of those prices which certainly has a great influence on the economically efficient use of the water in the reservoirs to produce that form of energy. The restrictions are determined not only by economical reasons of producing electricity, but also by ecological reasons and other uses of the reservoir water by the nearby population.

Consider an example of a two reservoir system shown in Figure 3.

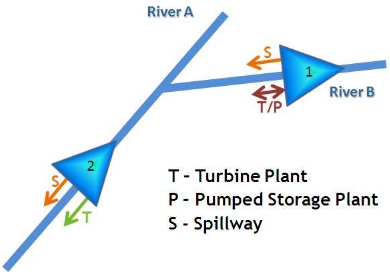

Figure 3: Two cascade reservoirs.

The optimization problem has the form

$$
\begin{aligned}
&P(q,s,V_0)=\int_0^T price(t)\left(\sum_{i=1}^{2}r_i(t)\right)dt\to\max,\\
&V_1(t)=V_1(t-1)+a_1-q_1(t)-s_1(t),\\
&V_2(t)=V_2(t-1)+a_2-q_2(t)-s_2(t)+q_2(t)+s_2(t),
\end{aligned}
$$

$$
\begin{aligned}
&V_i(0) = V_i^{in} \;\; i = 1,2,\\
&h_1(t) = Z_1^0 + \alpha_1 \left(\frac{V_1(t)}{V_1^0} - 1\right)^{\beta_1} - \max\left\{Z_2^0 + \alpha_2 \left(\frac{V_2(t)}{V_1^0} - 1\right)^{\beta_2}, \xi_1\right\},\\
&h_2(t) = Z_2^0 + \alpha_2 \left(\frac{V_2(t)}{V_2^0} - 1\right)^{\beta_2} - \xi_2,\\
&Z_i(t) = Z_i^0 + \alpha_i \left(\frac{V_i(t)}{V_i^0} - 1\right)^{\beta_i}, \; i = 1,2\\
&\zeta_1 \left(h_1(t) - h_1^0\right) - q_1^{0P} \le q_1(t) \le q_1^{0T} \left(h_1(t)/h_1^0\right)^{\frac{1}{2}},\\
&0 \le q_2(t) \le q_2^{0T} \left(h_2(t)/h_2^0\right)^{\frac{1}{2}},\\
&Z_i^{min} \le Z_i(t) \le Z_i^{max}, \; i = 1,2\\
&V_i^{in} - a_i \le V_i(T), \; i = 1,2.
\end{aligned}
$$

where $t = 1, 2, \ldots, T$.
In the next section we study this two reservoir system as well as a more involved four reservoir system shown in Figure 4. The main issue addressed in that section is the effectiveness of introduciong a turbining/pumping a link L between reservoirs 2 and 4.

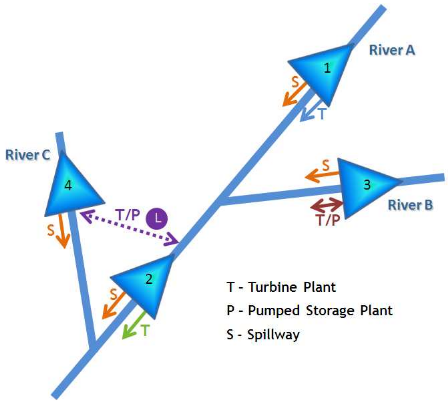

Figure 4: Four cascade reservoirs.

3 Computacional experience and results.

A computational experience with the two models was done in a hypothetical, but realistic, situation with real data of the water levels and flows, as well as the market prices of electricity.

The time period considered was one day, 24 hours, because of the great variability of intra-day electricity prices. Several type of days were tried, such dry, mildly wet and wet days, as well as different days of the week and days of different months. Only a sample of these results is presented.

The optimization problems were solved using a penalty function method. The problem had to be solved numerically because the complexity of the situation does not allow for an analytical solution to be found. In the case of two reservoirs the results are shown in Figure 5.
The calculations were done with the market prices of electricity shown in Figure 2. It should be noticed that the hydroelectric power stations associated with the two reservoirs only produce electricity when the price is high enough to justify that production. The system chooses to turbine the little water there is mainly at meals time. Nevertheless, as reservoir 1 is reversible, it turbines during a more enlarged period

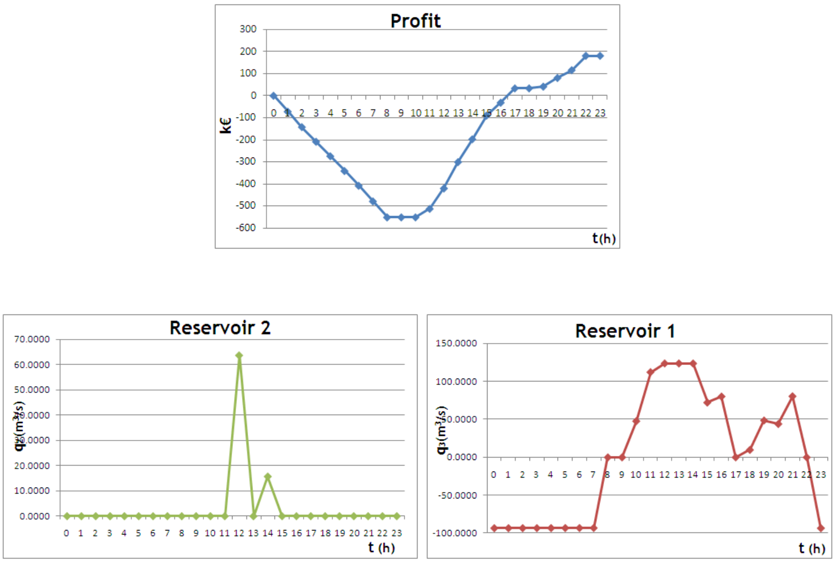

Figure 5: Example for two reservoirs.

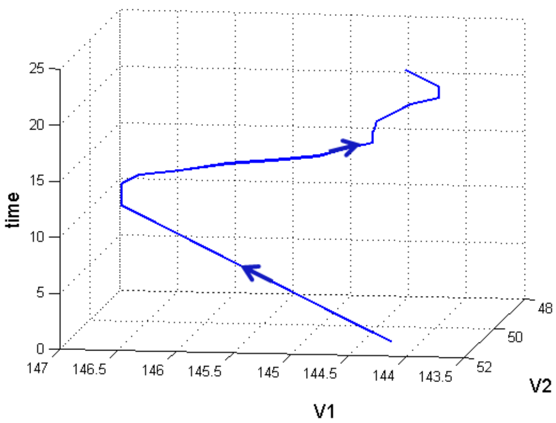

Figure 6: The optimal trajectory.

because even if the used volume of water exceeds the diary affluence, the excess is compensated at dawn when the price is lower.

It is interesting to see the optimal trajectory of the volume of water in the reservoirs which in Figure 6.

For the more complex cascade of four reservoirs, again with the same day market prices for the electricity, the obtained results are presented in Figure 7. It can be noticed a similar behaviour as in the previous case: electricity is produced when high prices justify the production. Now, reservoirs 3 and 4 are reversible and because of that water is pumped at dawn as in the previous case.

We also consider an intuitive water management model, that is, all the water is used to produce electricity when its price reaches the highest value and pumping is the option when the price is low enough, allowing later to use a bigger volume of water for production. The results with this intuitive policy are presented in Figure 8. For a 24 hour period the profit obtained using the "optimal" policy, was 255348.32 € and the profit with the intuitive policy was 136033.05 €.

The study was also done when link L (see Figure 4) was not present. The results are summarized in

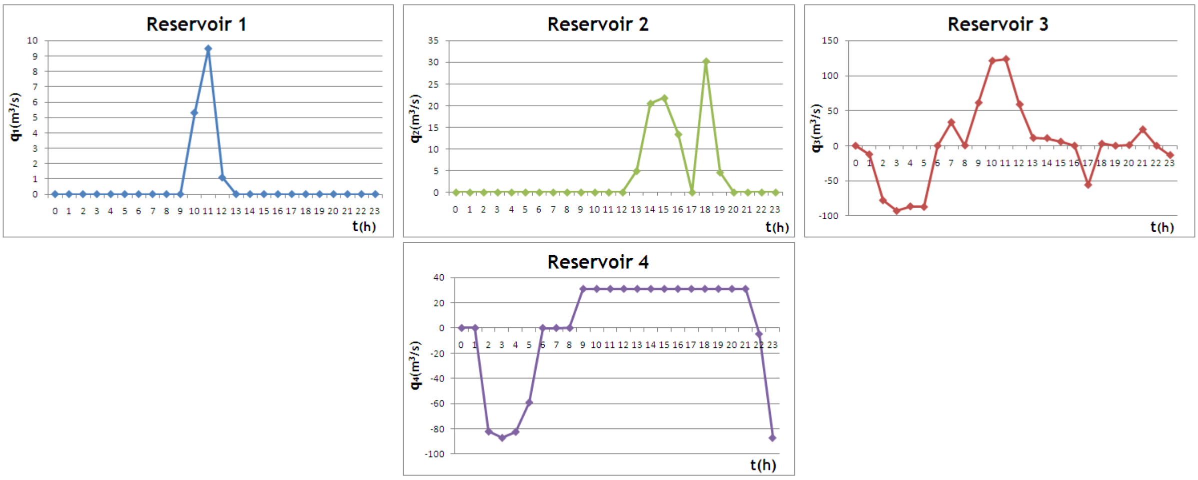

Figure 7: Four cascade reservoirs.

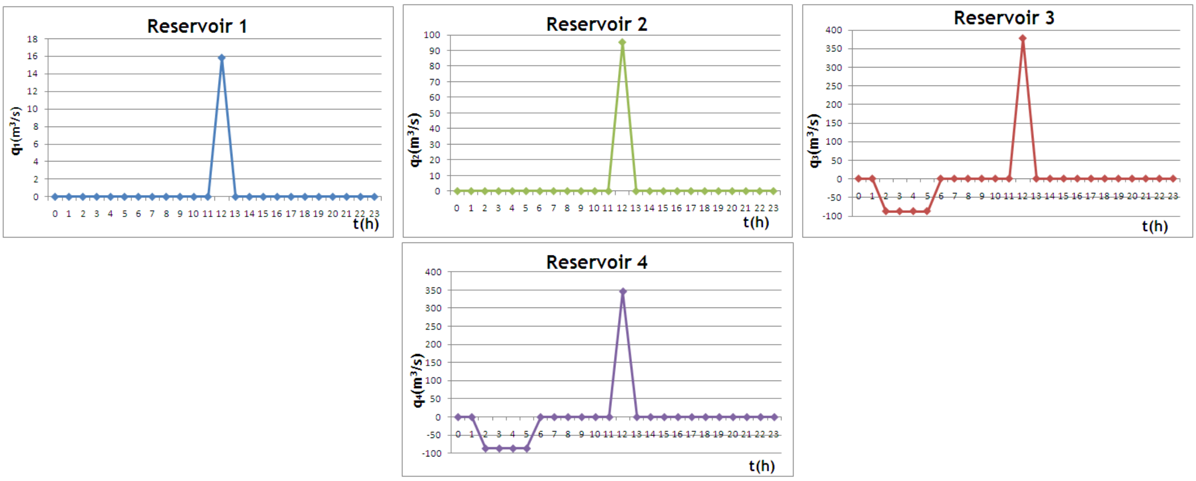

Figure 8: A simple policy and four cascade reservoirs.

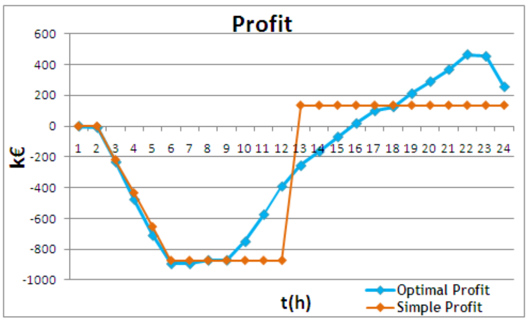

Figure 9: Comparison with a simple policy.

the following table:

		Wet	Average	Dry
	Cascade Inflow (m^3)	555.6	277.8	95.2
With link L	Profit (k€)	387.6	359.1	261.9
	Turbined Flow (m^3)	1095.9	1277.2	1140.8
	Pumped Flow (m^3)	892.6	832.3	1001.6
Without link L	Profit (k€)	329.7	320.7	112.5
	Turbined Flow (m^3)	1102.2	924.5	785.8
	Pumped Flow (m^3)	713.0	535.3	646.6

The profit in case with link L, with two possible situations of pumping and turbining to the same reservoir, has better values than in the case where link L is out, even if there is no lack of water. For a dry day, the profit obtained with link L has approximately doubled the one without link L. Since the water to be managed by the system is very little and as such, the inclusion of a reversible reservoir is essential to its reuse. For a wet day, the disposable water is enough. Since the level of water in each reservoir is nearer of the maximum admissible level, it is more difficult to manage the water and the situation becomes less flexible. Anyway, the link is advantageous because the system continues to reuse the water of the reservoir 2 having always a bigger profit.

We can than conclude that the inclusion of a reversible reservoir is always advantageous, and it shall be as more advantageous as less water the system has, that is, as far away is the volume of water from its upper limit.

4 Conclusions

A hypothetical situation of a cascade of hydroelectric power stations was considered with a possibility of turbining and pumping in some of the power stations. This is translated into a discrete-time optimal control problem which is solved numerically. The data used was real which gave a realistic situation. The developed model can be used to plan and to manage cascade power stations.

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