Machine Learning and PDEs

DM-FCUP, October 12, 2022

- Two mini-courses on recent applications of partial differential equations in Machine Learning.
- Structure: 1h lecture + 1/2h coffee break + 1h lecture + 1/2 h discussion.
- The material will be relatively self-contained and **accessible to undergraduate second and third year students**, and graduate students (Masters and PhD). Needless to say, everyone is welcome!

Mini-course # 1: 9:00–12:00

Room FC1 005

From Calculus of Variations to Reinforcement Learning

Diogo Gomes (KAUST)

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Abstract

This course begins with a brief introduction to classical calculus of variations and its applications to classical problems such as geodesic trajectories and the brachistochrone problem. Then, we examine Hamilton-Jacobi equations, the role of convexity and the classical verification theorem. Next, we illustrate the lack of classical solutions and motivate the definition of viscosity solutions. The course ends with a brief description of the reinforcement learning problem and its connection with Hamilton-Jacobi equations.

References

[1] M. Bardi and I. Capuzzo-Dolcetta, *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations. With appendices by Maurizio Falcone and Pierpaolo Soravia*, Systems & Control: Foundations & Applications, Birkhäuser Boston, Inc., Boston, MA, 1997.

[2] L. C. Evans, *Partial differential equations*, Graduate Studies in Mathematics 19, American Mathematical Society, Providence, RI, 1998. xviii+662 pp.

[3] D. Gomes, *Viscosity solutions of Hamilton-Jacobi equations*, Publicações Matemáticas do IMPA, 27o Colóquio Brasileiro de Matemática, Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, 2009. ii+210 pp.

Mini-course # 2: 14:00–17:00

Room FC1 031

Semi-Supervised Learning and the ∞-Laplacian

José Miguel Urbano (KAUST and CMUC)

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Abstract

Motivated by a recent application in Semi-Supervised Learning (SSL), the mini-course is a brief introduction to the analysis of infinity-harmonic functions. We will discuss the Lipschitz extension problem, its solution via MacShane-Whitney extensions and its several drawbacks, leading to the notion of AMLE (Absolutely Minimising Lipschitz Extension). We then explore the equivalence between being absolutely minimising Lipschitz, enjoying comparison with cones and solving the infinity-Laplace equation in the viscosity sense.

References

[1] J. Calder, *Consistency of Lipschitz Learning with infinite unlabeled data and finite labeled data*, SIAM J. Math. Data Sci. 1 (2019), 780–812.

[2] G. Aronsson, M. G. Crandall and P. Juutinen, *A tour of the theory of absolutely minimizing functions*, Bull. Amer. Math. Soc. (N.S.) 41 (2004), 439–505.

[3] M. G. Crandall, *A visit with the ∞-Laplace equation*, Calculus of variations and nonlinear partial differential equations, 75–122, Lecture Notes in Math., 1927, Springer, Berlin, 2008.

[4] P. Lindqvist, *Notes on the Infinity Laplace Equation*, SpringerBriefs in Mathematics, Springer, 2016.

[5] J.M. Urbano, *An introduction to the ∞-Laplacian*, Universidade de Coimbra, 2017.