

Seminar on Semigroups, Automata and Languages

Dilworth-Perkins construction and 14 limit varieties of aperiodic monoids

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Abstract: If W is a set of words in the alphabet \mathfrak{A} , then $M(W)$ (Perkins construction) denotes the Rees quotient of free monoid \mathfrak{A}^* over the ideal consisting of all words that are not subwords of words in W . A similar construction $S(W)$ for semigroups is attributed to Dilworth. The syntactic monoid $M_{synt}(W)$ is the quotient of \mathfrak{A}^* over the syntactic congruence \sim_W (the largest congruence for which W is a union of congruence classes). We generalize Dilworth-Perkins construction and syntactic monoids $M_{synt}(W)$ (resp. semigroups $S_{synt}(W)$) into $M_\tau(W)$ (resp. $S_\tau(W)$) construction, where τ is a congruence on \mathfrak{A}^* and $W \subseteq \mathfrak{A}^+$ is a union of τ -classes.

A variety of algebras is called *finitely based* (FB) if it has a finite basis of its identities. A minimal non-finitely based variety is called *limit*.

The first two explicit examples of limit varieties of monoids were found by Jackson in 2005. These varieties are generated by $M(\{abtasb, atbsab\})$ and $M(\{atbasb\})$. In 2019, Zhang and Luo found the third limit variety. Shortly after that, Gusev found a new pair of limit varieties \mathbb{J} and the dual of \mathbb{J} . We noticed that \mathbb{J} is generated by $M_\lambda([atbasb]_\lambda)$ or by the syntactic monoid $M_{synt}([atbasb]_\lambda)$, where $[atbasb]_\lambda = \{atba^m sb^n \mid m, n \geq 1\}$ is a congruence class of certain congruence λ . Together with Sergey Gusev we identified monoid of the form $M_\tau(W)$ which generates Zhang-Luo limit variety. It happens that this set of words W contains ab^2ta and atb^2a . These two words instructed Sergey and me on where to find five more limit varieties of monoids.

In this talk we use $M_\tau(W)$ construction to explain why Zhang-Luo variety is limit and to define five new limit varieties. We also identify syntactic monoids which are equationally equivalent to some finite monoids with interesting new properties discovered by Jackson and Lee (2015), Gusev and Vernikov (2017) and Gusev, Li and Zhang (2021).

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