CENTRO DE MATEMÁTICA UNIVERSIDADE DO PORTO

ALGEBRA AND GEOMETRY MINI-COURSE

The Weil conjectures: did André dream of étale sheaves?

João Lourenço

Université Sorbonne Paris Nord (Paris 13)

Abstract. In 1949, André Weil proposed four bold conjectures about the number of solutions of polynomial equations over finite fields, whose formulation revolves around the properties of a certain zeta function associated to the corresponding algebraic variety. In that very same paper, Weil simultaneously provided a proof for curves, building on his theory of Jacobian varieties, and a sketch of how the general case should follow from a cohomology theory that was yet to be constructed. Around 1963–64, Grothendieck built the étale cohomology theory Weil had envisioned, which allowed him to prove all but the third conjecture — known as the Riemann hypothesis, on account of its relation to the absolute values of the zeros of the zeta functions. One would have to wait a further ten years for Deligne to prove the last conjecture, via an ingenious induction using the celebrated Lefschetz pencils. This series of three talks will recount the history of these developments: from Gauss sums and Artin's zeta functions, through Poincaré duality and Weil's abelian varieties, to Grothendieck's étale cohomology. Time permitting, we may also sketch Deligne's ideas. Along the way, we will see how concepts from algebraic geometry, algebraic topology, and number theory culminated in one of the most spectacular theorems of twentieth-century mathematics.

Schedule (one lecture of 1h and two lectures of 1h30m):

- Wednesday, April 22, 14:00–15:00, room FC1-0.31.
- Friday, April 24, 14:00–15:30, room FC1-0.06.
- Tuesday, April 28, 09:30–11:00, room FC1-1.07.

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