

Dynamical Systems Seminar

Date. June 21, 11h30 (UNUSUAL TIME)

Place. Room FC1.031

Speakers. Jihoon Lee¹ (Department of Mathematics Sungkyunkwan University, Korea)

Title. Gromov-Hausdorff stability of reaction diffusion equations with Neumann boundary conditions under perturbations of the domain

Abstract. Let Ω_0 be an open bounded domain in $\mathbb{R}^N$ ($N \geq 2$) with C^1 boundary, and consider the reaction diffusion equations with Neumann boundary conditions

$$\begin{cases} \partial_t u - \Delta u = f(x, u) & \text{in } \Omega_0 \times (0, \infty), \\ \frac{\partial u}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega_0 \times (0, \infty), \end{cases} \quad (1)$$

where $\mathbf{n}$ is the outward unit normal vector field on the boundary $\partial\Omega$. Here we assume that $f : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function such that for any $x \in \mathbb{R}^N$, $f(x, \cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is C^2 and f satisfies the *dissipative condition*, i.e.,

$$\lim_{|s| \rightarrow \infty} \frac{f(x, s)}{s} < 0 \quad \text{uniformly in } x \in \mathbb{R}^N.$$

In this talk, we analyze how the asymptotic dynamics of the evolutionary system (1) changes when we vary the domain. Our first task is then to find a way to compare the global attractors of system (1) in different regions. One of the difficulties in this direction is that the phase space $H^1(\Omega_0)$ of the induced semi-dynamical system changes as we change the domain Ω_0 . In fact, the phase spaces $H^1(\Omega_0)$ and $H^1(\Omega_\varepsilon)$ which contain global attractors $\mathcal{A}_0$ and $\mathcal{A}_\varepsilon$, respectively, can be disjoint even if Ω_ε is a small perturbation of Ω_0 .

To overcome the difficulties, we use the Gromov-Hausdorff distance between two global attractors (which belong to disjoint phase spaces) to consider the continuity of global attractors and the Gromov-Hausdorff stability of system (1) under perturbations of the domain.

Finally we prove the continuity of global attractors and the Gromov-Hausdorff stability of reaction diffusion equations with Neumann boundary conditions under perturbations of the domain if every equilibrium of the system is hyperbolic.

¹Jihoon Lee is a Professor of Mathematics in Department of Mathematics Sungkyunkwan University (Korea). His research interests are: Nonlinear Partial Differential Equations, Mathematical Physics.